Bayesian Touch – A Statistical Criterion of Target Selection with Finger Touch

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ABSTRACT
To improve the accuracy of target selection for finger touch, we conceptualize finger touch input as an uncertain process, and derive a statistical target selection criterion, *Bayesian Touch Criterion*, by combining the basic Bayes' rule of probability with the generalized dual Gaussian distribution hypothesis of finger touch. The Bayesian Touch Criterion selects the intended target as the candidate with the shortest *Bayesian Touch Distance* to the touch point, which is computed from the touch point to the target center distance and the target size. We give the derivation of the Bayesian Touch Criterion and its empirical evaluation with two experiments. The results showed that for 2-dimensional circular target selection, the Bayesian Touch Criterion is significantly more accurate than the commonly used Visual Boundary Criterion (i.e., a target is selected if and only if the touch point falls within its boundary) and its two variants.

Categories and Subject Descriptors
H5.2 [Information interfaces and presentation]: User Interfaces. - Graphical user interfaces.

Keywords
Finger Touch; Target Selection; Bayes' Rule

INTRODUCTION
Finger touch has been widely adopted as a major input modality for various computing devices. It is direct, intuitive, always-available, and easily extensible to multi-touch. It has become the default interaction modality for Post-PC computing devices such as smartphones and tablets, and is even being integrated on regular PCs (e.g., touchscreen laptops).

Using finger touch to select a target among a set of candidates (target selection) is one of the most basic, important and frequently-performed finger touch tasks. However, it also suffers from the obvious "Fat Finger" problem. For smaller targets on the screen, it lacks the necessary precision to hit the intended one every time.

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Numerous methods have been proposed to improve the accuracy of target selection with a bare finger. Most of them attempted to map the input to the *intended touch point*. For example, Holz and Baudisch [10] proposed a method of estimating the intended touch point based on the 3D angle of the finger; Weir et al. [5] proposed mapping the raw sensor data or the touch location reported by the device to the intended touch point based on the historical touch behavior of a specific user. These techniques improve the touch accuracy by various degrees. However, employing them on current products would require either extra sensors [10] to measure finger postures, or knowing the specific user ID and her historical touch data [5].

Different from the previous research, we address the target selection problem with another approach. Instead of attempting to map finger touch to a single intended point, this approach conceptualizes finger touch input as an uncertain and ambiguous process. We keep the reported touch point unchanged, but devise statistical algorithms of deciding the intended target among distractors from the uncertain finger touch.

Current devices, by default, follow the *Visual Boundary (VB)* criterion to decide a target for a selection task: a target is selected if and only if the touch point falls within its boundary. Our experiment shows that this is a very erroneous criterion. Previous research [10] also reveals that to keep the error rate below 4%, the target size needs to be at least 4.3 mm wide if using this criterion.

The VB criterion matches the real word analogy of physical buttons with clear and deterministic edges. The underlying assumption is that the contact point is the point of interest and has to be within the visible boundary of the target However, this may not be the best analogy to finger touch on a purely visual target.

To reflect the inherent uncertainty of finger touch, we propose a statistical criterion of finger touch target selection — the *Bayesian Touch Criterion (BTC*). We derive *BTC* by treating the touch event as a statistical signal of the user's intention and inferring the probability of the intention accordingly. Specifically this decomposes to two steps. One is applying the Bayes' rule to estimate the probability of each object being the intended target given a signal. The other is to use the dual Gaussian distribution principle recently discovered about finger touch (Bi, Li and Zhai [4]). By combining an absolute precision component and a relative (to target size) precision component we built a Gaussian distribution of a given target's touch point distribution. These two steps lead to BTC, which serves as a principled criterion for ranking target likelihood.

BTC computes the *Bayesian Touch Distance (BTD)* between candidate targets and the touch point, and selects the candidate with the shortest BTD as the most probable target. The BTD between the touch point s and the target candidate t is:

$$BTD(s,t) = \frac{(s-c)^2}{2(\alpha W^2 + \sigma_a^2)} + \frac{1}{2}\ln(\alpha W^2 + \sigma_a^2) \qquad \text{Eq. 1}$$

where s is the finger touch coordinates reported by the device, W is the width of t, c is the center of t, α and σ_a are constants which can be measured via a separate experiment.

We conducted two experiments to show that BTC significantly reduced the error rate of target selection by 70% over VB, and by 26% and 6% over its two other variants, respectively. The accuracy improvement was achieved only using the touch point reported by device, without any extra sensing information.

RELATED WORK

Understanding Finger Touch

As a key input modality for touch screens, touch input has been extensively studied by many researchers. Wang and Ren [13] studied the properties of five fingers for touch input. Their results showed that the touch points followed bivariate Gaussian distributions, with the standard deviation varying across fingers. The touch points tended to spread wider for thumb and little fingers than index, middle and ring fingers. Azenkot & Zhai [1] and Henze et al.[9] studied the patterns of a special class of target selection tasks: entering text on a touchscreen keyboard. Both studies showed that touch points followed bivariate Gaussian distributions over the intended target key, often with more than 10% of touch points falling off the target key on a phone-sized device. The means of the Gaussian distributions were close to the key centers, but often with a small bias in different directions depending on hand posture (index finger vs. thumb), and regions of the keyboard [1]. Bi, Li and Zhai [4] proposed the dual Gaussian distribution hypothesis to interpret the distribution of touch points for Fitts' tasks [6], a special type of target selection tasks. They further derived the Finger-Fitts law (FFitts law) based on that hypothesis. Their study results validated the hypothesis and showed that the FFitts model had stronger predictive power than the conventional form of Fitts' law [6], which only models one of the two distributions in the dual Gaussian hypothesis.

Building on the literature, we apply the dual Gaussian distribution principle [6] to model the distribution of touch points based on the size and location of the target candidate, and combine it with the Bayes' rule to improve touch selection accuracy.

Improving Touch Accuracy

As target selection plays a critical role for touchscreen interaction, a sizable amount of research has been conducted to improve its accuracy.

Holz and Baudisch's [10] research showed that the offsets of touch point locations from the intended point were affected by the angles between the finger and the touch surface (i.e., pitch, roll and yaw). Their research showed that the accuracy could be substantially improved if the offset was compensated according to the user and posture. Such information can be obtained by either 3D tracking or scanning the fingerprint. In the following studies [11], they discovered that users relied on the visual features of fingers such as finger outlines and nail outlines for placing the touch point. Their findings suggested that the pointing accuracy could be substantially improved by tracking fingers with cameras from above.

Weir et al. [5] proposed a machine learning approach for learning a function that mapped an input (the devices' reported touch location or the raw sensor value) to the intended touch location. The study results showed that using the intended touch location substantially improved the accuracy.

Henz et al. [8] used a game published at the Android Market to collect a large number of touch events (more than 120 million). Based on the collect data, they trained a function that shifted the touch events to reduce the error rates. Evaluated by an updated version of the game, the trained function can significantly reduce the error rates.

Most of the methods proposed in the literature required extra information in addition to the touch location reported by device (e.g., 3D posture of the input finger [10, 11], or a user's touch history [5]). Distinguished from the previous research, the Bayesian Touch Criterion improves the touch interaction accuracy with the reported touch location only, which could be easily implemented on current touch screens without adding extra sensors or obtaining any prior user data.

Handling Input with Uncertainty

Since modern input modalities (e.g., finger touch) are usually ambiguous, researchers have proposed handling these inputs as uncertain events. Williamson [14] proposed framing the input as a continuous control process. In his view, the system a user interacts with continuously infers a distribution over potential user goals, and provides continuous feedback about its beliefs as it does so. Schwarz et al. [12] proposed carrying the uncertainty of input forward all the way through in the interaction, and deciding the final action via a mediation process. Complementary to Williamson's [14] and Schwarz et al.'s [12] work in general frameworks for handling uncertain inputs, we focus on a specific input modality: finger touch. We treat the finger touch as an uncertain process, and apply probabilistic theory to improve input accuracy.

BAYESIAN TOUCH

A Bayesian View of Target Selection

Target selection problems can be formulated as follows:

Let $T = \{t_1, t_2, \dots, t_n\}$ be the n target candidates on a touchscreen. Given a touch point s, the conditional probability that t ($t \in T$) is the intended target is $P(t|s)$. Determining the intended target is equivalent to finding t^* that maximizes $P(t|s)$:

$$t^* = \underset{t}{\operatorname{argmax}}\, P(t|s) \qquad \text{Eq. 2}$$

From Bayes' rule, $P(t|s)$ can be computed as:

$$P(t|s) = \frac{P(s|t)P(t)}{P(s)} \qquad \text{Eq. 3}$$

where $P(t)$ denotes the prior probability of selecting t without the observation of s, $P(s|t)$ is the likelihood function which expresses how probable the touch point s is if t is the intended target, and $P(s)$ is the normalization constant.

Since $P(s)$ is a constant, we have:

$$t^* = \underset{t}{\operatorname{argmax}}\, P(t|s) = \underset{t}{\operatorname{argmax}}[P(s|t)\, P(t)] \qquad \text{Eq.4}$$

Because the distribution of the touch point is Gaussian, the likelihood function can be written in the form (for 1-dimensional target):

$$P(s|t) = P(s|\mu, \sigma^2) = \frac{1}{(2\pi\sigma^2)^{1/2}} \exp\{-\frac{1}{2\sigma^2}(s-\mu)^2\} \quad \text{Eq. 5}$$

where s is the location of the touch point, μ and σ^2 are the mean and variance of the touch point distribution if t is the intended target.

As a common practice, maximizing the likelihood function is converted to minimizing the negative log likelihood function, which is in the form

$$-\ln P(s|t) = \frac{1}{2\sigma^2}(s-\mu)^2 + \ln\sigma + \frac{1}{2}\ln(2\pi) \qquad \text{Eq.6}$$

From Eqs. 4, 5 and 6, we have;

$$\begin{aligned} t^* &= \underset{t}{\operatorname{argmax}}[P(s|t)\, P(t)] \\ &= \underset{t}{\operatorname{argmin}}[-\ln P(s|t) - \ln P(t)] \\ &= \underset{t}{\operatorname{argmin}}\left[\frac{1}{2\sigma^2}(s-\mu)^2 + \ln\sigma + \frac{1}{2}\ln(2\pi) - \ln P(t)\right] \\ &= \underset{t}{\operatorname{argmin}}\left[\frac{1}{2\sigma^2}(s-\mu)^2 + \ln\sigma - \ln P(t)\right] \qquad \text{Eq. 7} \end{aligned}$$

$P(t)$ can be estimated from user history or design expectations (some targets are expected to be used more often than others). It is usually dependent on the application. In text input, for example, such information may come from language modeling or its simplest form, a lexicon [15]. Some letter keys are more likely than others given a context. In the most conservative and most unfavorable case of no prior knowledge about where the intended target is, we can assume that every candidate in T has an equal probability to be selected. Under this assumption, Eq 7 becomes:

$$t^* = \underset{t}{\operatorname{argmin}}(\frac{1}{2\sigma^2}(s-\mu)^2 + \ln\sigma) \qquad \text{Eq. 8}$$

We define the Bayesian Touch Distance (BTD) as:

$$BTD(s,t) = \frac{1}{2\sigma^2}(s-\mu)^2 + \ln\sigma \qquad \text{Eq. 9}$$

Eqs 8 and 9 show that finding the target t^* given a touch point s is equivalent to finding a value for t^* that minimizes the Bayesian Touch Distance. If candidates are not equally probable prior to s, $-\ln P(t)$ should be added to $BTD(s,t)$. Using $BTD(s,t)$ to decide the selected target is also referred as the *Bayesian Touch Criterion (BTC)* hereafter. In this paper we focus on the general target selection tasks in which we assume each candidate has an equal prior probability.

The Dual Gaussian Distribution Hypothesis

To compute $BTD(s,t)$, we need to know s, μ and σ. The touch point s is reported by the device. In this section, we propose an approach to estimate μ and σ from the size and location of target t. The method is derived from the dual Gaussian distribution hypothesis.

The dual Gaussian distributions hypothesis was initially introduced and verified by Bi, Li, and Zhai [4]. It considers the distribution of touch points (X) as a sum of two independent Gaussian distributions X_r, and X_a. Formally, this hypothesis can be expressed as:

$$X = X_r + X_a \sim N(\mu, \sigma^2) \qquad \text{Eq. 10}$$

μ and σ^2 are:

$$\mu = \mu_r + \mu_a \qquad \text{Eq. 11}$$

$$\sigma^2 = \sigma_r^2 + \sigma_a^2 \qquad \text{Eq. 12}$$

X_r is extrinsic to the user's motor control limit and relative to target properties (particularly size). The variance of X_r results from the speed-accuracy tradeoff of the user. It reflects to which degree the user has chosen to comply with the task precision. It is independent of the precision of the selection implement (e.g., pen or finger).

X_a is intrinsic to the motor control system, reflecting the absolute precision of a cursor, pen, or finger-based input method. It is independent of the user's desire to follow the specified task precision. X_a is negligible in a cursor-based input method, but significant in finger touch-based methods, particularly when the finger width is wider ("fatter") than the target width.

In the current work, we generalize the dual Gaussian distribution hypothesis from Fitts' tasks—which are special target selection tasks involving both amplitude (A) and target width (W)—to the more general target-selection tasks which are predominantly characterized by W alone. In tasks such as successive target selection with a single pointer, A is well-defined. In other tasks, such as alternating two thumb typing, or a single finger touch from a resting position off the screen, A is less well-defined.

Similar to the form in Fitts' tasks, we hypothesize that the touch point distribution X, if t is the intended target, is a sum of two independent Gaussian distributions: X_r and X_a.

Unlike Fitts' tasks in which the desired task precision is specified by A/W (A is the movement amplitude and W is the width of t), the precision is specified solely by W in target selection tasks.

In accordance with the underpinning of Fitts' law, at least in its original form, W affords the degree the touch points are expected to spread out. If the task precision is exactly complied with, no more, no less, we can expect $W \approx 4.133\sigma_r$, which means 96.4% touch points fall within the target. In practice a user might tend to over/under utilize W.

Note that it is σ_r instead of σ that is proportional to W. The reason is that X_a is irrelevant to the precision of the task, and should be removed when considering to what degree the user decides to comply with the task precision.

Assuming that σ_r is proportional to W, we have

$$\sigma_r^2 = \alpha W^2 \qquad \text{Eq. 13}$$

where α is a constant.

Replacing σ_r^2 in Eq. 12 with Eq. 13, σ^2 can be computed as

$$\sigma^2 = \sigma_r^2 + \sigma_a^2 = \alpha W^2 + \sigma_a^2 \qquad \text{Eq. 14}$$

Note that both α and σ_a^2 are constants and can be measured independently of the task. Eq. 14 shows that the variance (σ^2) is linearly correlated to the target size W^2, and the linear regression between σ^2 and W^2 has a non-zero intercept on σ^2-axis due to the low absolute precision of finger touch (σ_a^2).

Previous research [1, 8, 10] has shown that μ may have a small offset from the center of t (denoted by c). However, the magnitude and direction of the offset vary with individuals [10], finger vs. thumb [1], finger angle [10] and other factors. Across these factors, it is reasonable to expect the central tendency as

$$\mu \approx c. \qquad \text{Eq. 15}$$

Eqs 14 and 15 essentially predict μ and σ from the size and center of t. Replacing μ and σ in Eq. 9 with Eqs 14 and 15, we have

$$BTD(s,t) = \frac{1}{2\sigma^2}(s-\mu)^2 + \ln\sigma$$

$$= \frac{(s-c)^2}{2(\alpha W^2 + \sigma_a^2)} + \frac{1}{2}\ln(\alpha W^2 + \sigma_a^2), \qquad \text{Eq. 16}$$

where s is touch point location, W is width of t, c is the center of t, α and σ_a are constants.

Eq. 16 shows that $BTD(s,t)$ is computed based on the square distance of the touch point to the center of the target, the size of the target, and the finger absolute touch precision σ_a^2.

Two-Dimensional Circular Target Selections

With some simplifications, we generalize *BTC* to 2-dimensional circular targets. The derivation shows *BTC* for 2-dimensional circular target selection tasks is equivalent to finding the target that minimizes $BTD_2(s,t)$, the 2-dimensional Bayesian touch distance between the touch point s and the target candidate t.

Touch Points Distribution for 2-Dimenstional Circular Targets. Previous research shows that touch points approximately follow a bivariate Gaussian distribution for a 2-dimensional target selection [1, 8, 13].

$$X \sim N(\mu, \Sigma) \qquad \text{Eq. 17}$$

The correlation (ρ) between x and y coordinates of the touch points is small, but varies depending on the locations of the targets, and could be either negative or positive [1, 8]. Across various locations, it is reasonable to expect $\rho \approx 0$. μ and Σ of the distribution then become:

$$\mu = \begin{pmatrix}\mu_x \\ \mu_y\end{pmatrix} \qquad \text{Eq. 18}$$

$$\Sigma = \begin{pmatrix}\sigma_x^2 & 0 \\ 0 & \sigma_y^2\end{pmatrix} \qquad \text{Eq. 19}$$

where μ_x and μ_y are means, and σ_x and σ_y are standard deviations in x and y directions respectively.

According to the definition of bivariate Gaussian distribution, the negative log likelihood function becomes:

$$-\ln P(s|\mu,\Sigma) =$$

$$\frac{1}{2}\left[\frac{(s_x-\mu_x)^2}{\sigma_x^2} + \frac{(s_y-\mu_y)^2}{\sigma_y^2}\right] + \ln\sigma_x + \ln\sigma_y + \ln(2\pi), \qquad \text{Eq. 20}$$

where s_x, and s_y are coordinates of the touch point s.

$BTD_2(s,t)$, the generalization of $BTD(s,t)$ (Eq. 9) for 2-dimensional circular targets, becomes

$$BTD_2(s,t) = \frac{1}{2}\left[\frac{(s_x-\mu_x)^2}{\sigma_x^2} + \frac{(s_y-\mu_y)^2}{\sigma_y^2}\right] + \ln\sigma_x + \ln\sigma_y \qquad \text{Eq. 21}$$

2-Dimensional Dual Gaussian Distributions Hypothesis. To compute $BTD_2(s,t)$, we generalize the dual Gaussian distribution hypothesis to 2-dimensional circular targets.

Similar to the 1-dimensional targets, we hypothesize that the distribution of touch points for a 2-dimensional circular target is the sum of two independent Gaussians: X_r and X_a.

$$X_r \sim N\left(\begin{pmatrix}\mu_{r_x}\\ \mu_{r_y}\end{pmatrix}, \begin{pmatrix}\sigma_{r_x}^2 & 0\\ 0 & \sigma_{r_y}^2\end{pmatrix}\right) \quad \text{Eq. 22}$$

$$X_a \sim N\left(\begin{pmatrix}\mu_{a_x}\\ \mu_{a_y}\end{pmatrix}, \begin{pmatrix}\sigma_{a_x}^2 & 0\\ 0 & \sigma_{a_y}^2\end{pmatrix}\right) \quad \text{Eq. 23}$$

where μ_{r_x} and $\sigma_{r_x}^2$ are the mean and variance of X_r in the x direction, μ_{r_y} and $\sigma_{r_y}^2$ for X_r in the y direction, μ_{a_x} and $\sigma_{a_x}^2$ for X_a in the x direction, and μ_{a_y} and $\sigma_{a_y}^2$ for X_a in the y direction. Since we expect $\rho \approx 0$ for X, it is logical to expect $\rho_r \approx 0$ and $\rho_a \approx 0$ for X_r and X_a, respectively.

Similar to the 1-dimensional tasks, we assume both σ_{r_x} and σ_{r_y} are proportional to the size of t (the diameter d), and the μ is close to the center c (c_x , c_y) of t. The touch point distribution (X) of t can be expressed as:

$$X \sim N\left(\begin{pmatrix}u_x\\ u_y\end{pmatrix}, \begin{pmatrix}\sigma_x^2 & 0\\ 0 & \sigma_y^2\end{pmatrix}\right)$$

$$= N\left(\begin{pmatrix}c_x\\ c_y\end{pmatrix}, \begin{pmatrix}\sigma_{r_x}^2 + \sigma_{a_x}^2 & 0\\ 0 & \sigma_{r_y}^2 + \sigma_{a_y}^2\end{pmatrix}\right)$$

$$= N\left(\begin{pmatrix}c_x\\ c_y\end{pmatrix}, \begin{pmatrix}\alpha_x d^2 + \sigma_{a_x}^2 & 0\\ 0 & \alpha_y d^2 + \sigma_{a_y}^2\end{pmatrix}\right) \quad \text{Eq. 24}$$

Replacing μ_x, μ_y, σ_x and σ_y in Eq. 21 with the estimations in Eq 24, we have:

$$BTD_2(s,t) = \frac{1}{2}\left[\frac{(s_x - \mu_x)^2}{\sigma_x^2} + \frac{(s_y - \mu_y)^2}{\sigma_y^2}\right] + \ln\sigma_x + \ln\sigma_y$$

$$= \frac{1}{2}\left[\frac{(s_x - c_x)^2}{\alpha_x d^2 + \sigma_{a_x}^2} + \frac{(s_y - c_y)^2}{\alpha_y d^2 + \sigma_{a_y}^2}\right] + \frac{1}{2}\ln\left(\alpha_x d^2 + \sigma_{a_x}^2\right)$$

$$+ \frac{1}{2}\ln\left(\alpha_y d^2 + \sigma_{a_y}^2\right) \quad \text{Eq.25}$$

Note that α_x, σ_{a_x}, α_y, and σ_{a_y} are constants and can be measured via a separated experiment.

After generalizing the dual Gaussian distribution hypothesis to the 2-dimensional circular targets, $BTD_2(s,t)$ can be computed based on touch point location s (s_x, s_y), center c (c_x, c_y) and diameter (d) of t.

EXPERIMENT 1. MODELING TOUCH INPUT

Theoretically, BTC builds on Bayes' theorem and the dual Gaussian distribution hypothesis. In many steps of derivation we made simplifications and assumptions. Fundamentally, we applied mathematical (quantitative) reasoning to human behavior. The reliability of such a process should not be taken for granted without empirical confirmation. In the rest of the paper we seek experimental verifications of BTC.

Though the original dual Gaussian distribution hypothesis has been verified by Bi, Li and Zhai [4] in tasks where participants selected a target with a certain size (W) over a distance (A), we first conducted an experiment (Expt. 1) to verify the generalized dual Gaussian distribution hypothesis for 2-dimensional circular target selection tasks where the task precision is solely specified by the target size W.

More specifically, Expt. 1 is to verify the prediction made by the generalized dual Gaussian distribution hypothesis (Eq. 24): variances (i.e., σ_x^2 and σ_y^2) are linearly correlated to the target size d^2, and have non-zero intercepts on the variance-axes due to the low absolute precision of finger touch (i.e., $\sigma_{a_x}^2$ and $\sigma_{a_y}^2$),

We also aimed to estimate α_x, σ_{a_x}, α_y, and σ_{a_y} via Expt. 1, which would be used to compute $BTD_2(s,t)$ for other target selection tasks.

Participants and Apparatus

We recruited 18 participants (3 females) between the ages of 26 and 49. Two of them were left-handed. All of them used touchscreen devices (e.g., smartphones) several times a day. The experiment was conducted on a Galaxy Nexus phone running Android OS 4.2. The capacitive touch screen was 11.94 cm in diagonal with an aspect ratio of 9:16 and a resolution of 720 × 1280 pixels. When a finger touched the screen, the approximate centroid of contact area between the finger and the screen was reported as the touch point.

Tasks

Each participant performed a set of typical target selection tasks. At the beginning of each trial, a green circle appeared on the screen and the participant was instructed to select it using the input finger as quickly and accurately as possible. The target turned yellow and the device played a successful beep sound if the target was successfully selected, otherwise an error sound. The next trial started as soon as the input finger was lifted off. The selection of the target was determined by whether the touch point fell within the target, which is a common criterion for selecting UI elements on current touchscreen devices.

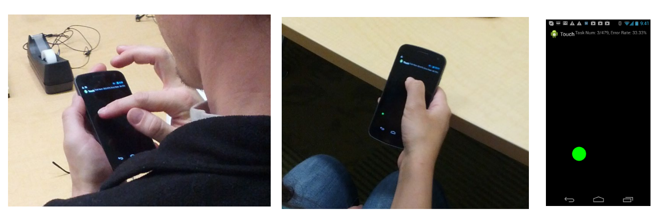
a b c

Figure 1. a: index finger posture. b: thumb posture. c: a target

Design

The study included 10 diameter (d) × input finger (f) conditions, with five levels of d (2, 4, 6, 8, 10 mm) combined with two levels of *f* (thumb and index finger). In the index finger condition, the participant acquired the target in a two-handed posture (Figure 1a), holding the device with one hand and acquiring the target with the index finger on the other. In the thumb condition (Figure 1b), the participant used a one-handed posture, holding it with one hand and using the thumb of the same hand to select it. Both postures are commonly used for interactions on touchscreen devices [1].

Each $d \times f$ combination included 80 trials. Trials with the same input finger (*f*) were grouped in a session. The order of trials in a session was randomized, as well as the location of the target in each trial. The order of sessions was counterbalanced across participants: half of the participant performed tasks with thumb first. A one-minute break was enforced before switching finger conditions.

In short, the study included:

18 (participants) × 2 (input finger) × 5 (target diameter) × 80 = 14400 trials.

Data Processing

We used the finger take-off position as the default location for a touch action. Trials with touch points 15 mm away from the target center were treated as outliers and discarded. The first 10 trials in each finger session of each user were treated as warm-up and were also excluded in the analysis. In total, 2% of trials were discarded and 14081 trials were analyzed.

Results

In agreement with previous research, the touch points followed bivariate Gaussian distributions for different target sizes. Shapiro-Wilk tests showed that touch points followed normal distributions for each user × target size ($p > 0.05$). Figure 4 shows three examples of touch point distributions.

To test the prediction from the dual Gaussian distribution hypothesis, we ran linear regressions for σ_x^2 vs. d^2, and σ_y^2 vs. d^2. Results are presented at Figure 2 & 3.

The regressions show strong linear relationships between the variance on each direction and d^2. The R^2 values are 0.93 for σ_x^2 vs. d^2 and 0.98 for σ_y^2 vs. d^2. Figure 2 & 3 also show non-zero intercepts on the y-axes.

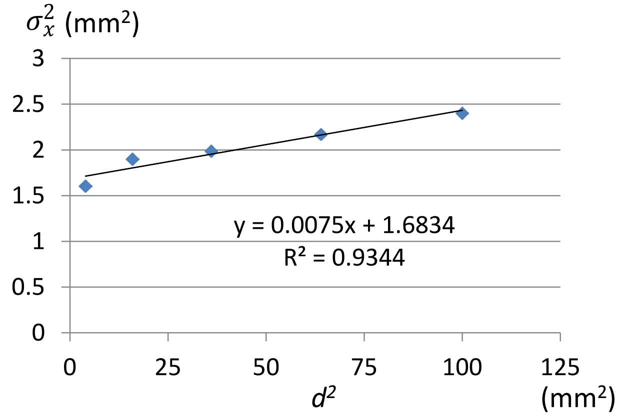

Figure 2. Regression between variance on *X* direction (σ_x^2) and target size (d^2).

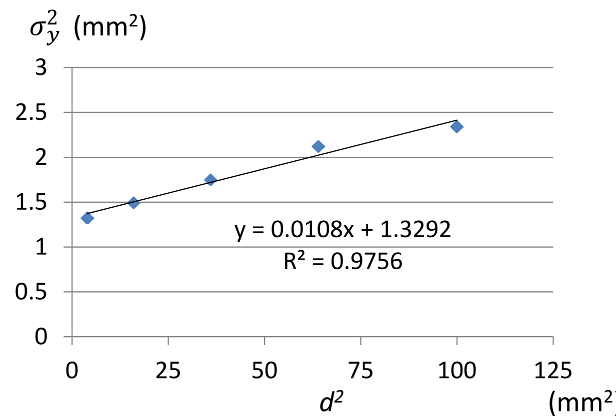

Figure 3. Regression between variance on *Y* direction (σ_y^2) and target size (d^2).

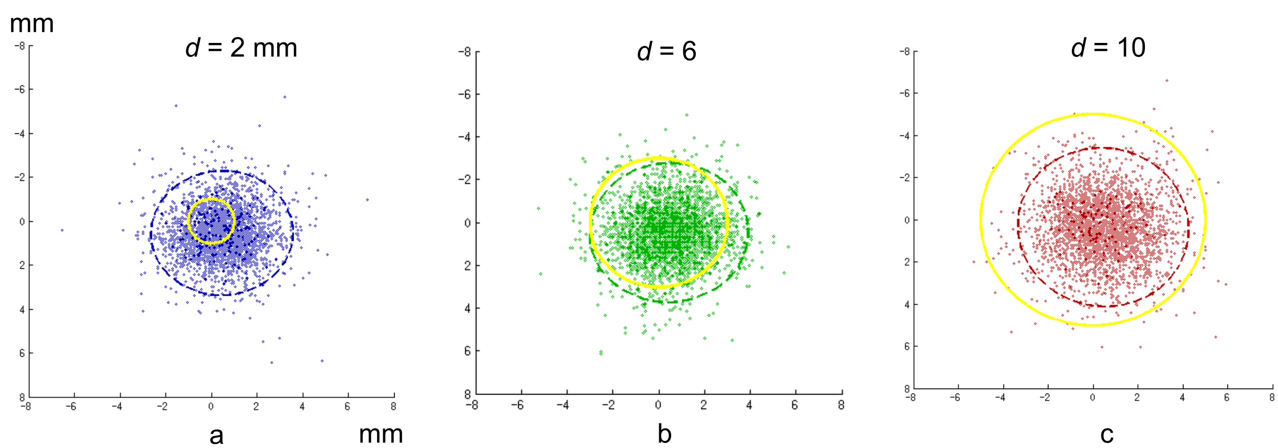

Figure 4. Distributions of touch points across all users for 3 target sizes. The yellow circles are the targets, and the dashed ellipses are 95% confidence ellipses. Touch point coordinates were calibrated with the origins at target centers.

The linear regressions provide the estimations of $\alpha_x, \sigma^2_{a_x}, \alpha_y,$ and$\sigma^2_{a_y}$, which are reported in Table 1.

α_x	$\sigma^2_{a_x}$	α_y	$\sigma^2_{a_y}$
0.0075	1.68	0.0108	1.33

Table 1. Constants $\alpha_x, \sigma^2_{a_x}, \alpha_y,$ and $\sigma^2_{a_y}$ computed from the linear regressions.

Replacing the constants in Eq.25 with the values in Table 1, $BTD_{2D}(s,\ t)$ can be computed as:

$$BTD_{2D}(s,t) = \frac{1}{2}\left[\frac{(s_x - c_x)^2}{\alpha_x d^2 + \sigma^2_{a_x}} + \frac{(s_y - c_y)^2}{\alpha_y d^2 + \sigma^2_{a_y}}\right]$$

$$+\frac{1}{2}\ln\left(\alpha_x d^2 + \sigma^2_{a_x}\right) + \frac{1}{2}\ln\left(\alpha_y d^2 + \sigma^2_{a_y}\right)$$

$$=\frac{1}{2}\left[\frac{(s_x-c_x)^2}{0.0075d^2+1.68} + \frac{(s_y-c_y)^2}{0.0108d^2+1.33}\right] + \frac{1}{2}\ln(0.0075d^2 + 1.68)$$

$$+\frac{1}{2}\ln(0.0108d^2 + 1.33) \qquad \text{Eq. 26}$$

Thumb vs. Index Finger. Previous research [13] showed that the index finger and thumb have different precision, with the index finger slightly more precise than the thumb. The different properties of the index finger and thumb might lead to differences in $\alpha_x, \sigma_{a_x}, \alpha_y,$ or σ_{a_y} for different fingers.

To evaluate this, we ran regressions for data in thumb and index finger conditions respectively. Results are reported in Table 2.

	α_x	σ_{a_x}	α_y	σ_{a_y}
Thumb	0.0073	1.35	0.0113	1.18
Index	0.0075	1.24	0.0104	1.12

Table 2. $\alpha_x, \sigma_{a_x}, \alpha_y,$ and σ_{a_x} for thumb and index finger specifically.

Table 2 shows that the differences in $\alpha_x, \sigma_{a_x}, \alpha_y,$ or σ_{a_y} between the thumb and index finger are small. The biggest difference is for σ_{a_x}, with the index finger being 8% lower than the thumb condition. Both σ_{a_x} and σ_{a_y} are slightly greater for the thumb than for the index finger, indicating that the thumb has lower absolute precision.

Discussion

The study results strongly concur with the prediction made by the generalized dual Gaussian distribution hypothesis. The study results showed R^2 of linear regression was 0.93 between σ^2_x and d^2, and 0.98 between σ^2_y and d^2, indicating strong linear relationships between variances and target size d^2. Figure 2 and Figure 3 also show non-zero intercepts on the y-axis.

The study results also provide estimations of constants $\alpha_x, \sigma_{a_x}, \alpha_y,$ and σ_{a_y}. With these estimations, we can use Eq. 26 to compute $BTD_{2D}(s,t)$ for other 2-dimensional circular target selection tasks.

EXPERIMENT 2. EVALUATING BAYESIAN TOUCH CRITERION

The purpose of the study is to evaluate *BTC* by comparing it with other target-deciding criteria.

Participants and Apparatus

Eighteen participants (7 females) between the ages of 21 and 38 participated in the study. One of them was left-handed. All of them used touchscreen devices (e.g., smartphones) several times a day. Importantly, to ensure the external validity of the model parameters estimated from Experiment 1, none of them participated in the Experiment 1. The apparatus was the same as that in Experiment 1.

Task

Each participant performed a set of target selection tasks. Unlike the task in Experiment 1, in which each trial displayed only one green circle as the target, the tasks in Experiment 2 showed multiple grey circles as distractors in addition to the green circular target (Figure 5). The participant was instructed to select the target (i.e., green circle) as accurately and quickly as possible.

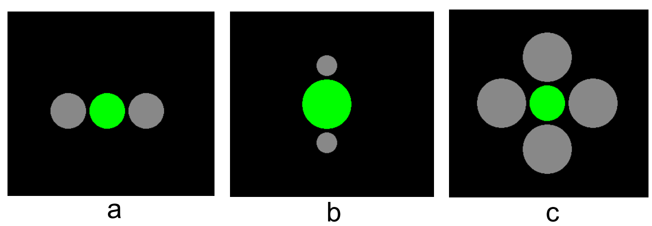

Figure 5. Three layouts of targets (green circles) and distractors (grey circles) used in the studies. The target and distractor sizes varied in the studies. The gap between the edge of the target and the edge of the distractor was 0.5 mm.

The task was designed to simulate the behaviors of UI elements on current touchscreen devices. If the touch point fell within a grey circle, the grey circle turned yellow and the device played an error sound, indicating a false selection. If the touch point did not fall within any circles, the device provided no feedback, indicating that nothing was selected. If the touch point fell within the green circle, the target turned green and the device played a successful sound. To encourage the participant to minimize errors, the participant needed to place the touch point within the boundary of the target (i.e., the *VB* rule) to continue to the next trial. Otherwise the trial repeated. The numbers of completed trials, remaining trials, and error rate were displayed at the right corner of the screen.

Design

The study included 54 target diameter (d_T) × distractor diameter (d_D) × distractor layout × input finger conditions.

Both d_T and d_D had 3 levels (3, 5, 7 mm), to represent small, medium, and large targets. Distractor layout also had 3 levels, which was illustrated in Figure 5. Input finger had 2 levels (thumb and index finger). Each condition had 20 repetitions with the location randomized each time. The process of the experiment was the same with that in Experiment 1.

In short, the experiment had:

$3d_T \times 3d_D \times$ 3 distractor layout × 2 input finger

× 20 trials × 18 participants = 19440 trials.

Note that some trials might repeat multiple times until the targets were successfully selected.

Target Selection Criteria

We designed and evaluated four target selection criteria:

1) The *Bayesian Touch Criterion (BTC)*. We use α_x, $\sigma_{\alpha_x}^2$, α_y, and $\sigma_{\alpha_y}^2$ measured from Experiment 1 (Table 1) to compute *BTD*. The candidate with the shortest *BTD* is selected.
2) *Visual Boundary (VB)*. A circle is selected if and only if the touch point falls within it.
3) *Visual Boundary or Shortest Distance to Circle Boundary (VB/SDB)*. It applies the *VB* rule first. If the touch point does not fall within any circle, the selected target is the circle with shortest distance to the touch point from its boundary. This criterion is also equivalent to the bubble cursor [7] and many snap-to-target techniques.
4) *Visual Boundary or Short Distance to Circle Center (VB/SDC)*. It applies the *VB* rule first. If the touch point does not fall within any circle, the selected target is the circle with the shortest distance to the touch point from its center. This criterion is less common than *VB* and *VB/SDB* in practice.

Criterion 1 (*BTC*) is derived from statistical principles as shown earlier. Criteria 2 – 4 are common sense criteria of target selection to which we want to compare *BTC*. The metric of evaluation for all of these criteria is selection *Error Rate*, defined as

$$Error\ Rate = \frac{Number\ of\ Failed\ Selections}{Total\ Number\ of\ Selections}$$

Data Processing

We removed outliers and the trials in warm-up according to the same criteria as Experiment 1. In total, 3% of all trials were removed and 24214 trials were analyzed.

Results

Error Rate. Figure 6 shows the overall error rate for each selection criterion. As shown, *BTC* had the lowest error rate. The mean error rates were 5.9% for *BTC*, 19.6% for *VB*, 8.0% for *VB/SDB*, 6.3% for *VB/SDC.* ANOVA showed the target selection criterion had a significant effect on error rate ($F_{3,\ 51}$ = 152, p < 0.001). Pairwise mean comparisons using the Bonferroni adjustment showed that *BTC* was significantly more accurate than all other criteria (p < 0.001).

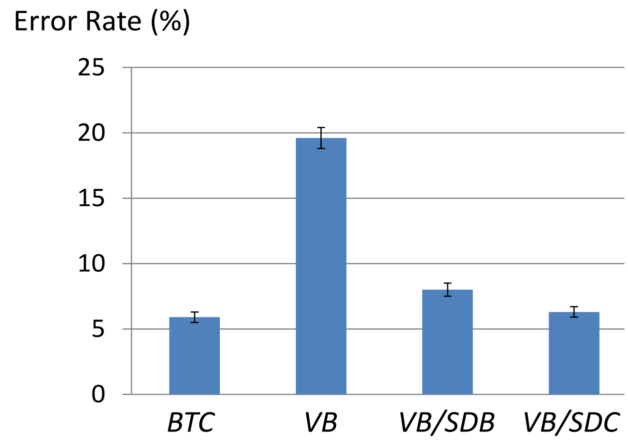

Figure 6. Mean and the standard error of mean (SEM) of Error Rate per target selection criterion.

Note that the *Visual Boundary (VB)* criterion resulted in the highest error rate, indicating that this commonly used criterion did not work well for finger touch under the experimental conditions tested.

ANOVA also showed that target diameter (d_T) ($F_{2,\ 34}$ = 110, p < 0.001), distractor diameter (d_D) ($F_{2,\ 34}$ = 33, p < 0.001), and distractor layout ($F_{2,\ 34}$ = 9.7, p < 0.001) had significant main effects on error rate. Smaller targets or distractors led to higher error rates. The mean error rates for 3, 5 and 7 mm target diameter (d_T) were 20.5%, 6.0%, and 2.1% , and for 3, 5 and 7mm distractor diameter (d_D) were 11.8%, 8.7%, and 8.0%, respectively. The layout in Figure 5(c) resulted in the highest error rate, 10.9%. ANOVA did not show a significant main effect of input finger on error rate ($F_{1,\ 17}$ = 0.818, p = 0.378).

ANOVA showed significant interaction effects for target diameter (d_T) × selection criteria ($F_{6,\ 102}$ = 314, p < 0.001) and distractor diameter (d_D) × selection criteria ($F_{6,\ 102}$ = 41, p < 0.001) on error rate. *BTC* outperformed other criteria with larger margins when targets were smaller or distractors were greater.

Touch Point Distributions. Since the participants in Experiment 2 were completely different from Experiment 1, it allowed us to evaluate the external validity of the dual Gaussian distribution hypothesis. We computed σ_x, and σ_y from the recorded data (i.e., *observed*), and compare them with the values predicted from Eq. 26 (i.e., *predicted*). We define prediction error as:

$$Prediction\ Error = \frac{|Predicted - Observed|}{Observed} \times 100\%$$

As shown in Table 3, Eq. 26 is able to closely predict σ_x and σ_y. The prediction error was below 7.5% across all the conditions. In particular, the error was 0 for σ_y when $d = 5$ and 7 mm.

d_T(mm)		σ_x	σ_y
3.0	Observed	1.23	1.26
	Predicted	1.32	1.19
	Prediction Error	7.3%	5.6%
5.0	Observed	1.35	1.27
	Predicted	1.37	1.27
	Prediction Error	1.5%	0
7.0	Observed	1.37	1.36
	Predicted	1.43	1.36
	Prediction Error	5%	0

Table 3. Prediction Error by Target Diameter (d_T)

To evaluate whether *BTC* performs better with finger-specific tuning, we computed *BTC* with α_x, $\sigma^2_{a_x}$,α_y, and $\sigma^2_{a_y}$ in Table 2 for index finger or thumb input respectively. The mean (and SEM) of overall error rate across participants was still 5.9% (0.4%), almost the same as not using finger-specific parameters. t-test did not show a significant difference ($p \approx 1.0$) between using generic parameters (Table 1) and using finger-specific parameters (Table 2).

Discussion

BTC outperforms VB and its variants. The study results verified that *BTC* was the most accurate criterion in selecting a user-intended target, providing strong support to the theoretical analysis leading to *BTC*. In particular, the error rate of *BTC* was 70%, and 26% lower than *VB* and *VB/SDB*, respectively.

The study results also verified the external validity of the dual Gaussian distribution hypothesis and showed its strong predictive power: σ_x and σ_y predicted by Eq. 26 closely matched the empirical results observed from the data in Experiment 2 (Table 3).

BTC can be adopted regardless of input finger. The study results showed that using finger-specific *BTC* did not lead to substantial performance improvement. It is probably because parameters (i.e., $\alpha_x, \sigma_{a_x}, \alpha_y$, and σ_{a_y}) do not vary drastically across fingers. The results show *BTC* can be adopted regardless of the input finger type.

LIMITATIONS AND FUTURE WORK

The current work is meant to be an exploration of a new and principled approach to finger touch target selection. We studied the Bayesian Touch Criterion at a general and fundamental level. Applying the criterion to specific and practical UI designs will require further verification and modifications. For example, we assumed each touch point is always meant for one of the objects on the screen. There is no "dead space" between objects. This of course does not have to be true. While *BTC* can be used as the primary criterion in ranking target likelihood, additional criteria can still be combined in a specific UI design problem. For example, additional thresholds and filters can be applied to the final target selection.

To maintain a manageable scope of two experiments in a single paper, we limited the target and distractor sizes to a range commonly seen on smartphone-sized devices. We see the very strong predictive power of the Bayesian Touch Criterion within this range of sizes.

CONCLUSION

To improve the accuracy of touch input, we conceptualize finger touch input as an uncertain and ambiguous process, and devise a statistical *Bayesian Touch Criterion (BTC)* for deciding the intended target among distractors. In addition to the theoretical analysis, we conducted experiments to verify our theory, and empirically evaluate *BTC*.

Our investigations lead to the following conclusions:

1) The *Bayesian Touch Criterion (BTC)* is a principled criterion for deciding the target for finger touch. It selects the intended target as the candidate with the shortest *Bayesian Touch Distance* (*BTD*) to the touch point. *BTD* is a function of touch point s and the target candidate t. For 1-dimensional tasks, it is in the form:

$$BTD(s,t) = \frac{(s-c)^2}{2(\alpha W^2 + \sigma_a^2)} + \frac{1}{2} ln(\alpha W^2 + \sigma_a^2) \qquad \text{Eq.27}$$

where s is the location of touch point, W is the width of t, c is the center of t, α and σ_a are constants which can be measured via a separate experiment.

For 2-dimensional circular targets, we not only derived the equation for computing *BTD*, but also measured the parameters via an experiment. Our investigation shows the 2-dimensional Bayesian touch distance $BTD_2(s,t)$ is in the form:

$$BTD_2(s,t) = \frac{1}{2}\left[\frac{(s_x-c_x)^2}{0.0075d^2+1.68} + \frac{(s_y-c_y)^2}{0.0108d^2+1.33}\right]$$
$$+\frac{1}{2}\ln(0.0075d^2 + 1.68) + \frac{1}{2}\ln(0.0108d^2 + 1.33) \text{ Eq. 28}$$

where s_x, s_y are the coordinates of touch point s, c_x and c_y are the coordinates of the center of a target candidate t, and d is its diameter. The units of these variables are mm.

2) The empirical experiment for 2-dimensional circular target selection shows that *BTC* is significantly more

accurate than the typical target deciding method, *Visual Boundary (VB)*, and its two variants: *Visual Boundary/Shortest Distance to Circle Boundary (VB/SDB)* and *Visual Boundary/Shortest Distance to Circle Center (VB/SDC)*. *BTC* reduces the error rate by 70%, 26% and 6% over *VB*, *VB/SDB*, and *VB/SDC* respectively.

3) *BTC* is a basis for touch UI design. *BTC* has a foundational role in researching and designing future touch user interfaces. Actual and specific touch UI designs may combine it with many other concerns and criteria of target selection. We believe fruitful future work should be expected along the direction this work points to.

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REFERENCES
1. Azenkot, S. and Zhai, S. (2012). Touch Behavior with Different Postures on Soft Smart Phone Keyboards. *Proc. of MobileHCI'12*. 251 - 260.
2. Baudisch, P. and Chu, G. (2009) Back-of-device interaction allows creating very small touch devices. *ACM CHI*, 1923-1932.
3. Benko, H., Wilson, A. D., and Baudisch, P. (2006) Precise selection techniques for multi-touch screens. *ACM CHI*, 1263-1272.
4. Bi, X., Li, Y., and Zhai, S. (2013) FFitts law: modeling finger touch with Fitts' law. *ACM CHI,* 1363 – 1372.
5. Weir, D., Rogers, S., Murray-Smith, R., and Löchtefeld, M., (2012). A user-specific machine learning approach for improving touch accuracy on mobile devices. *ACM UIST* 465-476.
6. Fitts, P.M. (1954). The information capacity of the human motor system in controlling the amplitude of movement. *Journal of Experimental Psychology*, 47, 381-391.
7. Grossman, T., and Balakrishnan, R. (2005). The bubble cursor: enhancing target acquisition by dynamic resizing of the cursor's activation area. *ACM CHI*. 281-290.
8. Henze, N., Rukzio, E., and Boll, S. (2011). 100,000,000 taps: analysis and improvement of touch performance in the large. *Proc. MobileHCI* , 133-142
9. Henze, N., Rukzio, E., and Boll, S. (2012) Observational and experimental investigation of typing behaviour using virtual keyboards for mobile devices. *ACM CHI.* 2659-2668.
10. Holz, C. and Baudisch, P. (2010) The Generalized Perceived Input Point Model and How to Double Touch Accuracy by Extracting Fingerprints. *ACM CHI*, 581-590.
11. Holz, C. and Baudisch, P. (2011). Understanding Touch. *ACM CHI,* 2501-2510.
12. Schwarz, J., Hudson, S., Mankoff, J. and Wilson, A. (2010). A framework for robust and flexible handling of inputs with uncertainty. *ACM UIST*. 47-56.
13. Wang, F. and Ren, X. (2009) Empirical evaluation for finger input properties in multi-touch interaction. *ACM CHI*, 1063-1072.
14. Williamson J. (2006) Continuous Uncertain Interaction. *Ph.D. Thesis*. University of Glasgow.
15. Zhai, S. and Kristensson, P. O. (2013) The word-gesture keyboard: reimagining keyboard interaction. *Commun. ACM* 55, 9 (September 2012), 91-101.